Computational Aspects of Geometric Correction Data Generation in the Landsat-D Imagery Processing.*

I. Levine
General Electric, Space Division
4701 Forbes Blvd., Lanham, MD 20706

ABSTRACT

A method is presented for systematic and geodetic correction data calculation. It is based on presentation of image distortions as a sum of nominal distortions and linear effects caused by variation of the spacecraft position and attitude variables from their nominals. The method may be used for both MSS and TM image data and it is incorporated into the processing by means of mostly offline calculations. Modeling shows that the maximal errors of the method are of the order of 5m at the worst point in a frame; the standard deviations of the average errors less than .8m.

INTRODUCTION

The geometric correction of the Landsat-type imagery typically proceeds in two steps. The first, called the Systematic Correction, removes internal distortions imported in the raw image data by the sensor mechanism, spacecraft motion, inaccurate sensor pointing, earth's rotation, etc. These partly corrected images still contain distortions due to uncertainties in spacecraft position and orientation. The second step, Geodetic Correction, removes these residual distortions using refined values of the attitude and ephemeris estimates. The refined attitude and ephemeris are obtained by filtering of image dislocations at Control Points.

Application of the geometric correction requires the generation of the Correction Data - Systematic (SCD) or Geodetic (GCD), depending upon the processing step. This data is developed on a rectangular grid in input (pixel, scan line) coordinates and express the relationship between the input and output map coordinates, within a standard World Reference System (WRS) frame.

The central part of the SCD/GCD generation is the computation of the coordinates of the intersection of the sensor's line-of-sight vector, with the Earth's surface (lookpoint coordinates). The lookpoint coordinates must then be converted to geodetic coordinates followed by mapping into user's map projection. There are two user's map projections: Space Oblique Mercator (SOM) and either Universe Transverse Mercator (UTM) or Polar Stereographic (PS).

* Work performed under National Aeronautics and Space Administration Contract No. NAS 5-25300.

Finally, the data, computed for integer values of pixels and lines, is interpolated to integer values of output map coordinates. The grid spacing is chosen so that the data, together with properly defined interpolation techniques, represent the output coordinates to the desired precision everywhere in the frame.

It should be noted that all the calculations are performed twice at each grid point, once for each SCD and GCD. They consume a significant amount of the processing time, which needs to be minimized. At the same time, there are no essential differences between SCD and GCD. Both establish a pointwise transformation, which may be written generically as

$$\overline{X}_m = F(\text{pixel}, \text{line}, \overline{p}),$$

where $\overline{X}_m = (X_{m1}, X_{m2})$ are map coordinates of a grid point and $\overline{p}$ is a vector of variables characterizing the spacecraft motion, attitude pointing, sensor's mechanism, etc.

Letting $\overline{p} = \overline{p}^n + \overline{\delta}$, the sum of nominal values of the variables and the deviation from the nominals, in the first approximation

$$\overline{X}_m = \overline{X}_m^{\,n} + \mu\,\overline{\delta}, \qquad (1)$$

where $\overline{X}_m^{\,n}$ are the nominal map coordinates and μ is the matrix of the partial derivatives (PD)

$$\mu = \left[\frac{\partial \overline{X}_m}{\partial \overline{\delta}}\right]$$

Thus, SCD and GCD may be represented as a sum of the nominal correction data and pointwise adjustments.

This has significant advantages:

1) It provides a uniform approach to the SCD and GCD computations, considering each as one transformation, and

2) Because the nominal spacecraft motion is known for every WRS frame, the nominal coordinates and the partial derivatives may be computed and stored in a Data Base.

The implementation of such an approach depends a great deal on both the choice of an output map projection and $\overline{\delta}$. An analytic form of mapping not only has to allow derivation of the coefficients μ, but it should also afford rapid and precise online inversion to geodetic coordinates, from which the final map projection can be generated. In addition, it is desirable to have the nominal coordinates and the partial derivatives, as far as possible, insensitive to global position of the frame. Thus, although out technique may be applied to most standard map projections (such as UTM or PS), a special intermediate projection, Local Space Oblique Mercator (LSOM), has been employed. The LSOM is the Mercator projection for the sphere, with local 'equator' along the nominal spacecraft inertial velocity at the frame center. In that projection $\overline{X}_m^{\,n}$ and μ are longitude-undependent and thus, can be stored only for one path. A natural choice of variables $\overline{\delta}$ is the along-track, cross-track and radial deviations in spacecraft position, together with deviations in the attitude angles. The nominal spacecraft motion within a frame is assumed to be in a perfect circular orbit passing through the frame center.

NOTATIONS

$\bar{a}_p = (a_{p1}, a_{p2}, \ldots, a_{pK})$	-	vector (Kx1 matrix)
$\|\|\bar{a}_p\|\|$	-	the Euclidean norm of $\bar{a}_p$
A^T	-	transposed matrics A
$\bar{X}$	-	spacecraft position vector in earth-centered earth-fixed coordinates
$\bar{X}_s$	-	spacecraft position vector in nominal spacecraft coordinates
$\bar{X}_m = (X_{m1}, X_{m2})$	-	output map coordinates
$\bar{g}^n$	-	pointing vector in body coordinates
$\bar{g}$	-	pointing vector in local vertical spacecraft coordinates
$\bar{f}$	-	pointing vector in earth-centered fixed coordinates
$\bar{u}$	-	coordinates of a lookpoint on the earth surface
h	-	distance from spacecraft to earth lookpoint
$\bar{v}$	-	$\bar{u} \cdot \|\|\bar{u}\|\|^{-1}$ - normalized lookpoint vector.
R	-	local earth radius at WRS frame center
Ω	-	earth rotation rate
a_e, b_e	-	earth equatorial and polar radii

$e_1 = e_2 = 1,\ e_3 = a_e = a_e b_e^{-1}$

δ_1 - deviation in the pitch

δ_2 - deviation in the roll

δ_3 - deviation in the yaw

δ_4 - along track angular deviation

δ_5 - cross track angular deviation

δ_6 - relative departure in the radial direction

δ_7 - time deviation

μ_k - (2x1) matrix of the partial derivatives of $\overline{X}_m$ with respect to δ_k.

$\mu = (\mu_1, \mu_2, \ldots, \mu_7)$

$\hat{\theta}_p$, $\hat{\theta}_r$ - the 'equivalent' pitch and roll

Ω_s - spacecraft orbital rate

λ - geodetic longitude

ϕ - geocentric latitude

$\hat{\phi}$ - geodetic latitude

$$ROT_1(\Psi) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{vmatrix}$$

$$ROT_2(\psi) = \begin{vmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{vmatrix}$$

$$ROT_3(\psi) = \begin{vmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\frac{\partial}{\partial\psi} ROT_i(\psi)_{\psi=0} = T_i$$

$$T_1 = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{vmatrix}$$

$$T_2 = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{vmatrix}$$

$$T_3 = \begin{vmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

I - the three dimensional identity matrix

The upper index η indicates the nominal value of a vector.

T_{act} - active scan time

T_{round} - mirror turnaround time

THE NOMINAL SCD

Coordinate Transformations

The local (instantaneous) spacecraft coordinates are described in terms of the unit vectors $(\bar{\xi}_1, \bar{\xi}_2, \bar{\xi}_3)$, where $\bar{\xi}_3$ points towards the Earth center, the $\bar{\xi}_1$ vector is along the orbital angular momentum, and $\bar{\xi}_2 = \bar{\xi}_3 \times \bar{\xi}_1$ is roughly along the velocity direction. The local spacecraft coordinates at the WRS center is called the nominal spacecraft coordinates. The matrix A transforms a vector $\bar{X}_o$ in earth-centered inertial coordinates to the vector $\bar{X}_s$ in nominal spacecraft coordinates:

$$\bar{X}_s = A\bar{X}_o \tag{2}$$

The inertial to earth-centered earth-fixed coordinate transformation is defined as

$$\bar{X} = E^T\bar{X}_o, \tag{3}$$

where $E = E_o \cdot ROT_3(\Omega t)$.

The matrix E_o gives the time-independent component of the transformation, $ROT_3(\Omega t)$ describes the rotation about the earth axis at the rate Ω. We assume that $t = 0$ at the frame center. The corresponding nominal spacecraft to earth fixed coordinate transformation may be written as

$$\bar{X} = E^T A^T \bar{X}_s \quad = P^T\bar{X}_s \tag{4}$$

where P = AE.

The unit vector $\bar{g}^n$, given in body (sensor) coordinates, is transformed to local spacecraft coordinates as

$$g = ROT_3(-\theta_y) \cdot ROT_2(-\theta_r) \cdot ROT(-\theta_p)\ \bar{g}^n \qquad (5)$$

where θ_y, θ_r, θ_p are the yaw, roll, and pitch.

In the nominal spacecraft coordinates, $\bar{g}_s$ may be expressed as

$$\bar{g}_s = ROT_1(\gamma)\bar{g}$$

where γ is the angle in the orbit plane between the spacecraft and the frame center. In the nominal spacecraft motion $\cos\gamma = -X_{s3}/\|\bar{X}_s\|$, $\sin\gamma = X_{s2}/\|\bar{X}_s\|$, and thus, the matrix $ROT_1(\gamma)$ is known completely.

A vector $\bar{X}_m = (X_{m1}, X_{m2})$ in LSOM coordinates is defined as

$$X_{m1} = \frac{R}{2}\ln\frac{1+\sin\beta}{1-\sin\beta} \qquad (7)$$

$$X_{m2} = R\alpha$$

where R is the earth local radius at the frame center. The local polar angles, α and β, are given by

$$W_1 = \sin\beta$$

$$W_2 = \cos\beta \cdot \sin\alpha \qquad (8)$$

$$W_3 = -\cos\beta \cdot \cos\alpha$$

where

$$\bar{W} = \|\bar{u}\|^{-1}\ A\bar{u} \qquad (9)$$

and $\bar{u} = (u_1, u_2, u_3)$ are earth fixed coordinates of the corresponding point on the ground.

Generation of the nominal SCD

The nominal coordinates, $\overline{X}_m$, are computed on a grid, consisting of $2n_1+1$ fictitious scan lines, each line containing $2n_2+1$ points. Because the TM scans in two directions, it requires two sets of the nominal coordinates, for forward and backward scans. The computations may be fulfilled in the following order.

1. Generate the time of (i,j) point

$$t_{ij} = \frac{T_{scene}}{2\ n_1}(i-n_1-1) + \frac{T_{act}}{2_{n2}}(j-n_2-1) + \Delta T$$

Here

$$T_{scene} = (T_{act} + T_{round}) \cdot (N_{scan} - K_1)$$

where T_{act} is the active scan time, T_{round} is the turnaround time, N_{scan} is the actual number of scans, and $K_1 = 1$ for MSS and 2 for TM.

The parameter $T = T_{scene}$ for backward scans of the TM and zero otherwise.

2. Generate $2n_2+1$ unit line-of-sight vectors $\overline{g}^n$ in body coordinates. An actual mirror velocity profile, together with constant sensor's misalignments may be employed.

3. Compute the spacecraft position vector $\overline{X}_s$ and the matrix P at t_{ij}.

4. Compute $\overline{g}_s$ according to (6).

5. Transform $\overline{X}_s$ and $\overline{g}_s$ in earth fixed coordinates, obtaining the vectors $\overline{X}$ and $\overline{f}$, respectively.

6. Determine the lookpoint coordinates, $\overline{u} = (u_1,u_2,u_3)$, and h from the equations

$$\overline{u} = \overline{x} + h\overline{f} \qquad (10)$$

$$(u_1^2 + u_2^2)\ a_e^{-2} + u_3^2\ b_e^{-2} = 1 \qquad (11)$$

7. Transform $\overline{u}$ into LSOM coordinates using (9), (8), and (7).

It is convenient to have all distances in units of the nominal orbit radius.

THE PARTIAL DERIVATIVES

Position and Pointing Vectors

Let $\overline{X}_s^n$ be a nominal spacecraft position vector at time t_o, δ_4 and δ_5 be the angular along-track and cross-track deviations in spacecraft position, and δ_6 be a relative deviation in the radial direction. Then the actual spacecraft position vector, $\overline{X}_s$, may be obtained by rotating $\overline{X}_s^n$ through δ_4 and δ_5. This is followed by stretching according to the ratio $1 - \delta_6$:

$$\overline{X}_s = ROT_2(\delta_5) \cdot ROT_1(\delta_4) \cdot \overline{X}_s^n(1-\delta_6)$$

Similarly, if δ_1, δ_2, and δ_3 are deviations in the pitch, roll, and yaw, the actual (unit) pointing vector $\bar{g}_s$ should be written as

$$\bar{g}_s = ROT_2(\delta_5) \cdot ROT_1(\delta_6)\bar{g}_s^n ,$$

where $\bar{g}_s^n$ is defined by (5) and (6) as

$$\bar{g}_s^n = ROT_1(\gamma)\, ROT_3(-\delta_3) ROT_2(-\delta_2) \cdot ROT_1(-\delta_1)\, \bar{g}^n$$

Let $t = t_o + \delta_7$. Remembering that $P = P(t_o) = AE_o \cdot ROT_3(\Omega t_o)$, we can write P^T at time t as

$$P^T(t) = ROT_3(-\Omega(t_o+\delta_7))E_o{}^T A^T = ROT_3(-\Omega\delta_7)\cdot$$

$$\cdot ROT_3(-\Omega t_o)E_o^T A^T = ROT_3(-\Omega\delta_7)P^T$$

and the actual position and pointing vectors in earth fixed coordinates as

$$\bar{X} = ROT_3(-\Omega\delta_7)P^T ROT_2(\delta_5)ROT_1(\delta_4)\bar{X}_s^n(1-\delta_6)$$

$$\bar{f} = ROT_3(-\Omega\delta_7)P^T ROT_2(\delta_5)ROT_1(\delta_4)ROT_1(\gamma)\cdot$$

$$ROT_3(-\delta_3) \cdot ROT_2(-\delta_2) \cdot ROT_1(-\delta_1)\bar{g}^n$$

The linear terms of the Taylor series expansions of $\bar{X}$ and $\bar{f}$ in the vicinity of $\delta_i = 0$ $(i = 1,2,\ldots,7)$ give

$$\bar{X} = P^T(I + T_1\delta_4 + T_2\delta_5 - \delta_6 - \Omega T_3\delta_7)\bar{X}_s^n$$

$$\bar{f} = P^T \Big[I + T_1 ROT_1(\gamma)\delta_4 + T_2 ROT_1(\gamma)\delta_5 - ROT_1(\gamma) \cdot \sum_{i=1}^{3} T_i\delta_i -$$

$$- \Omega \cdot ROT_1(\gamma)\delta_7 \Big] \bar{g}^n$$

Here we used the fact that

$$\frac{\partial}{\partial \psi} ROT_i(\psi)_{\psi=0} = T_i$$

Introducing $\bar{f}^n = P^T ROT_1(\gamma)\bar{g}^n$ and

$\bar{X}^n = P^T \bar{X}_s^n$, we finally have

$$\bar{X} = \bar{X}^n + P^T(T_1\delta_4 + T_2\delta_5 - \Omega T_3\delta_7)P\bar{X}^n - \bar{X}^n\delta_6$$

$$\bar{f} = \bar{f}^n + P^T(T_1\delta_4 + T_2\delta_5 - \Omega T_3\delta_7)P\bar{f}^n - \tag{12}$$

$$- P^T ROT_1(\gamma)\left(\sum_{i=1}^{3} T_i\delta_i\right)\bar{g}^n$$

The Partial Derivatives of Lookpoint Coordinates

Henceforth, we will use a prime to denote the matrix of PD with respect to δ computed at the nominal point. From (10) it follows that

$$M \quad \frac{\partial \bar{u}}{\partial \delta} = \bar{u}' = \bar{X}' + h\bar{f}' + h'\bar{f} \tag{13}$$

Introducing $e_1 = e_2 = 1$ and $e_3 = a_e b_e^{-1}$, Eq.(11) may be rewritten as

$$\sum_{i=1}^{3} e_i^2 (\bar{X}_i + h\bar{f}_i) = a_e^2$$

or,

$$h^2\left(\sum f_i e_i^2\right) + 2h\left(\sum f_i X_i e_i^2\right) + \sum X_i^2 e_i^2 = a_e^2$$

Differentiating the last expression as a inplicit function of h gives

$$h' = -\frac{\sum (X_i' + hf')(X_i + hf_i)e_i^2}{\sum f_i(X_i + hf_i)\ e_i^2} =$$

$$= -\frac{\sum u_i e_i^2 (X_i' + hf_i')}{\sum f_i u_i e_i^2}$$

and, after substitution h' in (13), we have

$$u_k' = C_o^{-1}\left[(X_i' + hf_k')\sum_{i \neq k} u_i f_i e_i^2 - f_k \sum_{i \neq k} (X_i' + hf_i')u_i e_i^2\right]$$

$(i,k = 1,2,3)$

where $\quad C_o = -\sum f_i u_i e_i^2$

Using the matrix notations, $\bar{u}^1$ may be expressed as

$$u^1 = C(\bar{X}^1 + h\bar{f}^1), \tag{14}$$

where C is a 3x3 matrix with the elements

$$C_{kk} = 1 + U_k f_k e_k^2 C_o^{-1}$$

$$C_{kj} = u_j f_k e_j^2 C_o^{-1} \tag{15}$$

Transformation to LSOM Coordinates

To transform the lookpoint coordinates, u, to the LSOM coordinates, they must be represented in the normalized form $\bar{V} = \bar{u} \cdot \|\bar{u}\|^{-1}$.

Differentiation of $\bar{V}$ Yields

$V_k^1 = (U_k^1 - U_k \|\bar{U}\|^2 \sum U_i U_i^1) \|\bar{U}\|^{-1}$ and introducing the matrix B with the elements

$$B_{kk} = 1 - U_k^2 \|\bar{U}\|^{-2} \tag{16}$$

$$B_{kj} = - U_k U_j \|\bar{U}\|^{-2},$$

$\bar{V}^1$ may be written as

$$\bar{V}^1 = \|\bar{U}\|^{-1} B\bar{U}^1 = \|\bar{U}\|^{-1} BC(\bar{X}^1 + h\bar{f}^1) \tag{17}$$

The next step is transformation of $\bar{V}$ to $\bar{W}$ and then to $\bar{X}_m$. From (9) and (17) it follows that

$$\bar{W}^1 = \|\bar{U}\|^{-1} ABC(\bar{X}^1 + h\bar{f}^1). \tag{18}$$

From (7) and (8) it follows that

$$X_{m1} = \tfrac{1}{2}R \cdot \ln \frac{1 + W_1}{1 - W_1}$$

$$X_{m2} = R \cdot \arctan (-W_2/W_3) \tag{19}$$

and therefore,

$$X_{m1}^1 = R(1-W_1^2)^{-1} W_1^1$$

$$X_{m2}^1 = R(1-W_1^2)(W_2 W_3^1 - W_3 W_2^1).$$

Introducing the matrix

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -W_3 & W_2 \end{pmatrix} \tag{20}$$

and using (18), we have

$$\overline{X}_m^1 = R(1-W_1^2)^{-1}\overline{W}^1 =$$

$$R \, \|\overline{U}\|^{-1}(1-W_1^2)^{-1} \, DABC(\overline{X}^1 + h\overline{f}^1) \tag{21}$$

To obtain the final result, we must substitute an explicit expression for $\overline{X}^1 + h\overline{f}^1$, which follows immediately from (12):

$$X_k^1 + hf_k^1 = \begin{cases} -hP^T ROT_1(\gamma) \; T_k \; \overline{g}^n & k = 1,2,3 \\ P^T T_{k-3} P(\overline{X}^n + h\overline{f}^n) = P^T T_{k-3} P\overline{U}^n & k = 4,5 \\ -\overline{X}^n & k = 6 \\ -\Omega T_3 \overline{U}^n & k = 7 \end{cases}$$

Description of the Algorithm

Calculation of the partial derivatives is performed simultaneously with the LSOM coordinate generation in the following order.

1. Compute matrices C,B, and D, given by (15), (16), and (20).

2. Compute matrices

$$J = \frac{R}{\|u\| \; (1-W_1^2)} \, DABC$$

$$J_o = JP^T$$

3. Compute vector $\overline{Z} = P\overline{u}^n$

4. Form five vectors

$$J_1 = \frac{h}{r}\,(0,\; X_{s2}g_2^r - X_{s3}g_3^n,\; X_{s2}g_3^n + X_{s3}g_2^n)$$

$$J_2 = \frac{-h}{r}\,(g_3^n r, g_1^n X_{s2}, g_1^n X_{s3})$$

$$J_3 = \frac{h}{r}\,(g_2^n r, g_1^n X_{s3}, -g_1^n X_{s2})$$

$$J_4 = (0, -Z_3, Z_2)$$

$$J_5 = (Z_3, 0, -Z_1)$$

$$J_7 = (U_2, -U_1, 0)$$

Here $r = \| \overline{X}_s \|$.

5. Compute

$$\mu_k = J_o J_k \qquad (k = 1,2,\ldots,5)$$

$$\mu_6 = -J\overline{X}^n$$

$$\mu_7 = \Omega J J_7$$

Here μ_k is the 2x1 matrix of the partial derivatives with respect to δ_k and thus,

$$\mu = (\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6, \mu_7).$$

Note, that SCD/GCD calculations require only the first six pair of the PD. The partial derivatives with respect to time, μ_7, will be used only to generate the backward scan grid for Thematic Mapper.

The nominal SCD and PD are computed in double precision and stored in single precision. Because the PD are changing very slowly over a frame, they may be computed on a sparse grid followed by linear interpolation onto a finer grid. For instance, implementation of our technique for MSS requires calculation of PD on a 3x5 grid.

THE NOMINAL COORDINATES AND THE PARTIAL DERIVATIVES FOR BACKWARD SCANS OF TM

It should be remembered that application of the developed technique to Thematic Mapper data requires two sets of the nominal SCD and PD - for forward and backward scans. But actually only one set must be obtained by the direct lookpoint calculation: LSOM coordinates for, say, forward scans may be easily converted to LSOM coordinates for backward scans. Our calculations show also that, for sensor's misalignments less than $.1^o$, the derivatives are practically same for both grids; for bigger misalignment the second set of the derivatives can be obtained by the linear interpolation of the first one.

Let $\overline{X}_m(t_1)$ and $\overline{X}_m(t_2)$ be LSOM coordinates for adjacent forward and backward scans at time t_1 and t_2, respectively. Note, that for the TM

$$\delta t = t_2 - t_1 \leq 2\, T_{act} + T_{round} = .132205 \text{ sec}$$

So, we will neglect changes in the attitude angles during δt.

Considering $\Delta X_m = \overline{X}_m(t_2)-\overline{X}_m(t_1)$ as a function of changes in the spacecraft position, sensor pointing, and effects of the earth rotation, we may represent it as

$$\Delta\overline{X}_m = \left(\frac{\overline{X}m}{\delta_4}\right)_{t_1} \Omega_s \delta t + \left(\frac{\overline{X}m}{\delta_1}\right)_{t_1} \hat{\theta}_p + \left(\frac{\overline{X}m}{\delta_2}\right)_{t_1} \overline{\theta}_r +$$

$$+ \left(\frac{\overline{X}m}{\delta_1}\right)_{t_1} \delta t = (\mu_4\,\Omega_s + \mu_7)\delta t + \mu_1\hat{\theta}_p + \mu_2\,\hat{\theta}_r$$

Here $\hat{\theta}_p$ and $\hat{\theta}_r$ are fictitious pitch and roll angles, reflecting a difference in sensor's pointing at t_1 and t_2, and Ω_s is the average orbital rate during δt.

Here we will denote the nominal pointing vector $\overline{g}^n$ at moments of time t_1 and t_2 as $\overline{P}$ and $\overline{q}$, respectively. The angle between their projections onto the (ξ_2,ξ_3) plane, $(0,P_2,P_3)$ and $(0,q_2,q_3)$, can be written as

$$\cos\hat{\theta}_p = \frac{P_2q_2 + P_3q_3}{(P_2^2 + P_3^2)^{\frac{1}{2}}(q_2^2 + q_3^2)^{\frac{1}{2}}}$$

or, choosing the appropriate sign,

$$\hat{\theta}_p \simeq \sin\hat{\theta}_p = \frac{P_3q_2 - P_2q_3}{(1-P_1^2)^{\frac{1}{2}}(1-q_1^2)^{\frac{1}{2}}}$$

Analogously, $\hat{\theta}_r$ may be expressed as the angle between projections of $\overline{P}$ and $\overline{q}$ onto the (ξ_1,ξ_3) plane:

$$\hat{\theta}_r \simeq \sin\hat{\theta}_r = \frac{P_1q_3 - P_3q_1}{(1-P_2^2)^{\frac{1}{2}}(1-q_2^2)^{\frac{1}{2}}}$$

For zero sensor's misalignments

$$\hat{\theta}_p \simeq -\frac{2\,P_2\,P_3}{(1-P_1^2)^{\frac{1}{2}}}$$

$$\hat{\theta}_r = 0$$

CONVERSION TO BASIC MAP PROJECTIONS

It should be remembered, that completely corrected imagery eventually must be presented in two basic map projections, SOM and either UTM or PS. To generate correction data in a basic map projection, it is required to invert LSOM coordinates to geodetic latitude and longitude and then perform the standard mapping into desirable projection.

Noting, that the normalized look point vector, $\overline{V}$, can be expressed through the geocentric latitude, ϕ, and the longitude, λ as

$$V_1 = \cos\lambda \cos\phi$$

$$V_2 = \sin\lambda \cos\phi$$

$$V_3 = \sin\phi \quad ,$$

and, employing well known formula for the geodetic latitude $\hat{\psi}$

$$\tan\hat{\phi} = a_e^2 b_e^{-2} \tan\phi,$$

one can obtain

$$\lambda = \arctan\left(V_2 V_1^{-1}\right)$$

$$\phi = \arctan\left[a_e^2 b_e^{-2} V_3 (1-V_3^2)^{-\frac{1}{2}}\right]$$

For a given $\overline{X}_m$, $\overline{V}$ is computed by the inverted formulae (10) and (9).

NUMERICAL RESULTS

The Accuracy of the Method

To evaluate the methods accuracy, differences between LSOM coordinates, computed directly on points of a grid, and those, corrected according to Eq.(1), were calculated for various spacecraft position and attitude deviations. It is convenient to characterize the upper level of errors by the maximal along-track (AT) and cross-track (CT) errors, which coorespond to the errors at the worst points of a frame (possible different for AT and CT errors). It should be noticed, that the maximal errors always appear near the corner points and similar for TM and MSS. They are linearly dependent upon magnitude of deviations and practically independent upon WRS latitude.

The actual position and attitude departures for Landsat-D are expected to be $.01^{\circ}(\sigma)$ for the pitch, roll, and yaw and less than 5km in the along and cross track directions. The radial departure is determined chiefly by the orbit fluctuations and it will not exceed 9.5km. Modeling shows, that for $\delta_1 = \delta_2 = \delta_3 = .03^{\circ}$, $\delta_4 = \delta_5 = 5$km, and $\delta_6 = 9.5$km, the corresponding maximal CT and AT errors have the order of 5m (CT = 4.87m, AT = 5.03m for MSS and CT = 4.97m, AT = 5.07m for TM). The inversion from LSOM to geodetic coordinates produces insignificant additional errors, therby preserving the same order of errors in UTM and PS projections.

For TM, the forward to backward scan conversion results in CT and AT errors less than .03m for zero sensor's misalignments; for extremely large misalignments of the order of $.5^{\circ}$, the maximal CT errors increase up to .5m.

Currently the SCD/GCD generation accuracy for Landsat-D are defined in terms of the average mean-squared errors (1m for TM and 1.5m for MSS). Table 1 represents the 90% maximal errors and the standard deviations of the average errors for Thematic Mapper, obtained by stochastic modeling. Here the attitude angles were normally distributed with zero means and $\sigma = .01^{o}$. Two cases of radial deviations were considered: a constant equal to 9.5 km, and a more plausible value from a uniform distribution (-9.5, 9.5) km. Because the errors do not depend significantly upon distribution of the cross and along track deviations, the latter were kept constant at 5 km. 400 samples were used to establish results for each case. The table also represents a case when PD were computed on a 3x7 grid and then interpolated to a finer grid. The nominal SCD and PD for backward scans were recomputed from the data for forward scans. Note, that in all cases the standard deviations of the average errors are less 1 m and thus, the nominal SCD and PD, precomputed for mean orbit radius at the frame center, provide the geometric correction with the required accuracy.

Timing

On the VAX, the direct lookpoint calculations take about 11 msec per grid point, interpolation of PD - 1 msec, the nominal SCD to SCD/GCD correction- less than .5 msec, and inversion from LSOM to geodetic coordinates - 1.1 msec per point. Application of our technique for MSS requires interpolation PD to a finer grid, two corrections in LSOM coordinates, and inversion to geodetic coordinates; altogether it takes about 3.1 msec per point. The direct on-line SCD and GCD calculation takes about 22 msec per point.

It should be noted, that mapping to the SOM requires about 15 msec per point, which is considered excessive for on-line processing. This time may be significantly reduced if we take into consideration the fact that the LSOM closely approximates true SOM distances between points within each frame. The errors of the approximation are relatively small (less than 5m) and sufficiently regular to permit linear interpolation LSOM to SOM coordinates. It may be done by using a 9x9 grid of corrections, precomputed and stored in the Data Base (Ref. 1).

CONCLUSIONS

The SCD/GCD calculation technique is based on presentation of image distortions as a sum of nominal distortions and linear effects, caused by variation of the spacecraft position and attitude variables from their nominals. The implementation requires generation and storage of the nominal SCD and twelve (for MSS), or fourteen (for TM) matrices of PD for each distinct latitude of WRS, along one path. The maximal errors of the method do not exceed 5.1m at the worst point of a frame. The standard deviations of the average errors are less than 1m. The speed of the processing and the accuracy that is achieved by this technique makes it an elegant solution in the production environment.

Table 1.

The 90% maximal errors and the standard deviations of the average errors for constant and uniformally distributed radial deviations.

Distribution of the radial deviation	Interpolation of PD	Forward scans				Backward scans			
		90% max errors (m)		STD (m)		90% max errors (m)		STD (m)	
		CT	AT	CT	AT	CT	AT	CT	AT
constant	no	2.76	2.37	.61	.49	2.78	2.36	.61	.49
	yes	3.09	2.69	.63	.67	3.11	2.70	.70	.72
uniform	no	1.31	1.78	.29	.32	1.33	1.78	.58	.58
	yes	2.19	1.41	.42	.35	2.19	1.40	.53	.48

REFERENCES

1. J. Brooks, R. Kumar, I. Levine, "Implementation of the Space Oblique Mercator Projection in a Production Environment", Fifteenth Symposium on Remote Sensing of Environment, May 11-15, 1981, Ann Arbor, Michigan.